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Deflection of Rectangular Orthotropic Plates Under Uniform Load

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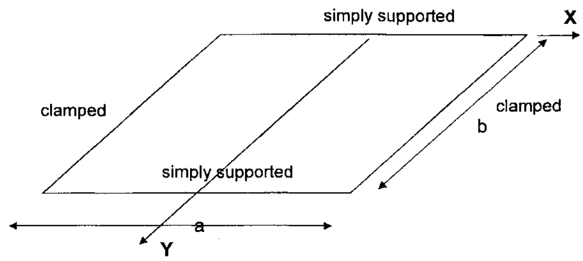

Fig. 1 Coordinate system for plate analysis.

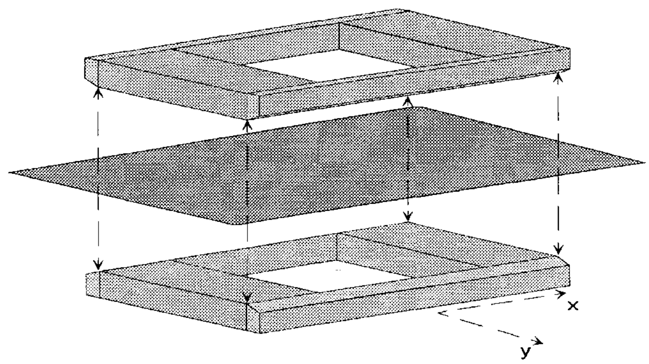

Fig. 2 Plate deflection text fixture.

Introduction

IT has been analytically shown that the maximum deflection of rectangular orthotropic plates under uniform transverse loading with symmetric edge conditions does not occur at the center for all aspect ratios (AR).[1] Depending on the AR, curvatures for orthotropic plates under uniform loading have more than one maximum and reverse sign.

The purpose of this Note is to validate the aforementioned theoretical results. An experiment was performed to determine the deflected shape of symmetrically supported orthotropic plates under uniform load, as shown in Fig. 1. The experimental results were then compared with the mathematical model for shape and deflection. A symmetric clamped simply supported boundary condition case was selected, because it has a larger difference in peak to overall plate deflection compared with a plate that is simply supported on all sides.

Experimental Apparatus and Procedures

Fixture Fabrication

A fixture was designed to apply transverse uniform loading to a rectangular clamped simply supported plate, as shown in Fig. 2. The fixture consists of four clamp blocks and four knife edges that are bolted together to form two symmetric halves that are 15.24 cm (6.00 in.) wide. The knife edges on the upper half of the fixture are aligned with the clamp block faces to provide even boundary conditions without distorting the plate. The lower knife edges, which also form the sides of an air chamber, or bladder, are spaced approximately 1.3 mm (0.05 in.) from the plate to allow free rotation. During the experiment, the bladder was inflated with air to provide a uniform load on the lower plate surface. The fixture was designed to work with three different AR (1.88, 1.00, and 0.62) by changing the position of the clamp blocks. This required disassembly and realignment of the fixture for each AR tested.

After the fixture was assembled and aligned, the bladder was formed across the lower half of the fixture using an aluminum base plate on the bottom and a thin elastic sheet of nylon film on the tops of the lower knife edges. The nylon film was held in place with room temperature vulcanizing silicone adhesive, which was also used to seal the rest of the joints in the fixture. The final assembly of the fixture consisted of bolting the test plate between the upper and lower fixture halves.

Plate Fabrication

Graphite/epoxy was chosen for plate construction, because it was readily available and produces an orthotropic laminate having a significant ratio of maximum deflection to overall deflection variation for this experiment. The plate was fabricated from 16 plies of Hercules AS-4/3501-6 graphite/epoxy prepreg, which was laid up and autoclave cured as recommended by the manufacturer. This resulted in a finished plate that was 1.83 mm (0.072 in.) thick.

Experimental Setup

The fixture was set up on the bed of a large vertical milling machine, which provided the accuracy and repeatability necessary to make consistent deflection measurements. A dial indicator was mounted to the head of the milling machine in a manner that allowed the indicator tip to contact the plate through the complete deflection range. After aligning the test fixture with the dial indicator, the bed of the milling machine was set up to track the y axis across the center of the plate, because this is where the greatest amount of deflection was expected. A set of zero load deflection measurements was taken in 2.54-mm (0.10-in.) increments across the width of the plate. This resulted in 58 measurements because of interference between the fixture edge and the indicator tip. Subsequent measurements were then taken with various amounts of pressure applied to the fixture to load the plate. Pressure was supplied by a precision air pump or regulated shop air line and was recorded during each test using gauges that were calibrated with a U-tube mercury manometer.

Numerical Analysis

The deflection $w(x, y)$, given by a Levy solution for a plate whose sides at $x = \pm a/2$ is

$$w_p(y) + \frac{b^4}{D_2\pi^4}\sum_{n=1}^{\infty}\frac{a_n}{n^4}\left[\frac{s_2\cosh(n\pi s_1 x/b)\sinh(n\pi s_2 c/2) - s_1\cosh(n\pi s_2 x/b)\sinh(n\pi s_1 c/2)}{s_1\cosh(n\pi s_2 c/2)\sinh(n\pi s_1 c/2) - s_2\cosh(n\pi s_1 c/2)\sinh(n\pi s_2 c/2)}\right]\sin\left(\frac{n\pi y}{b}\right)$$

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Table 1 Material properties used in the numerical solution

Material	E_1	E_2	ν_{12}	G_{12}
Graphite/epoxy	155 GPa	10 GPa	0.3	7 GPa

Table 2 Variation in maximum deflection relative to center deflection for AR = 0.6 plate

y axis location	27%	50%	73%	α
Theory	0.253 mm	0.246 mm	0.253 mm	2.8%
Experiment	0.245 mm	0.240 mm	0.255 mm	4.2%

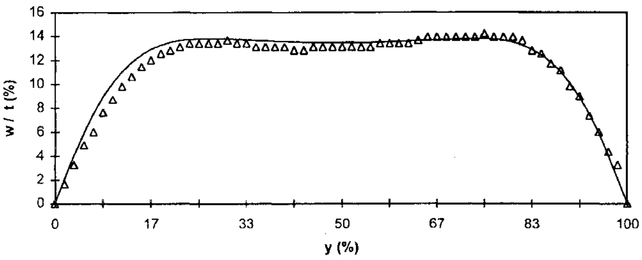

Fig. 3 Theoretical vs experimental plate deflection for AR = 0.62 graphite/epoxy unidirectional ply plate: ——, AR = 0.62 theory and △, AR = 0.62 experiment.

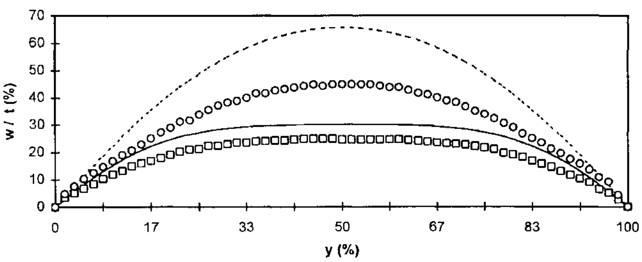

Fig. 4 Theoretical vs experimental plate deflection for AR = 1.00 and 1.88 graphite/epoxy unidirectional ply plate: ——, AR = 1.00 theory; - - - -, AR = 1.88 theory; □, AR = 1.00 experiment; and ○, AR = 1.88 experiment.

where s_1 and s_2 are the complex conjugate roots to the characteristic equation[1,2]

$$D_1 s^4 + 2D_3 s^2 + D_2 = 0$$

For a uniformly distributed load the particular solution is

$$w_p = (q_0/24D_2)(y^4 - 2by^3 + b^3 y)$$

with coefficients $a_n = (4q_0/\pi n)$ for $n = 1, 3, 5, \ldots$, and 0 for $n = 2, 4, 6, \ldots$.

The overall deflected plate shape and the deflected plate shape at $x = 0$ were calculated and plotted for the three AR tested. The material properties used in the numerical solution are shown in Table 1 (Ref. 1).

Results and Discussion

A comparison of the experimental data with the theoretical results given by the Levy solution was made both numerically and graphically. A variable α is defined as the variation in maximum deflection relative to the center deflection,[1]

$$\alpha = \frac{w_{\max} - w_{\text{center}}}{w_{\text{center}}} \times 100$$

where $w_{\max}$ is taken as the average local maxima of the plate.

Values for α are shown in Table 2 for the AR = 0.6 plate. The measured deflections were taken at the locations of theoretical local maxima because the experimental curve was not symmetric.

A graphical comparison of the theoretical and experimental results for the three AR tested is shown in Figs. 3 and 4. The shape of the theoretical plate deflection is close to the experimental data, which shows the predicted reverse curvature for AR < 1. The magnitude of theoretical deflection is conservative for the higher AR, which could not be explained by calculation or experimental measurement error. A brief study of the effect various parameters have on the numerical solution showed that it was possible to improve the accuracy of the maximum deflection results, while maintaining the proper curve shapes, by varying the material properties. The purpose of this short Note is to qualitatively validate theoretical predictions about the behavior of orthotropic plates; therefore the material property values given in Ref. 1 were used rather than experimentally obtained material properties. It is assumed that inaccurate material property values used in the numerical model, along with the inability to experimentally obtain theoretically perfect boundary conditions, is partly responsible for the deflection errors. However, the large difference between the theoretical and experimental deformations shown in Fig. 4 (AR = 1.00 and 1.88) is perhaps a result of the overprediction of deflection by small deformation plate theory. One may need large deformation theory to accurately predict deflections as large as one-half the plate thickness.

Conclusions

The experimental observations qualitatively confirm the analytically predicted behavior of orthotropic plates. This phenomenon of maximum deflection occurring at locations other than the center for orthotropic plates can be a significant factor in composite plate design.

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Eigenvector Derivatives with Repeated Eigenvalues Using Generalized Inverse Technique

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I. Introduction

DERIVATIVES of eigenvalues and eigenvectors are being used in the analysis for guidance in design modification and for improving analytical models in many technical fields.[1–5] The determination of eigenvalue derivatives can be a straightforward calculation, but the computation of eigenvector derivatives is very time consuming. The techniques of the calculation of eigenvector derivatives of a large complex eigensystem are problem dependent. The main reason is that the algebraic equations acquired upon differentiating the eigensystem relationships result in an undetermined system of equations. For the eigenvector derivative computation, the most simple approach is by finite difference method. Although the finite difference method is easy to implement into the computer program, in many cases the numerical perturbation step sizes and the selection of proper design parameters become even more expensive than other direct analytical methods.

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